Localized tendency for the superfluid and Mott insulator state in the array of dissipative cavities

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The features of superfluid-Mott insulator phase transition in the array of dissipative cavities is analyzed. Employing a kind of quasi-boson and a mean-filed approach, we show analytically how dissipation and decoherence influence the critical behaviors and the time evolution of the system. We find that there is a localized tendency, which could lead to the break of superfluidity for a superfluid state and suppress the appearance of the long-range order form a Mott state. Eventually, a collection of mixture states localized on each site will arise.

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One of the remarkable applications of coupled cavity arrays is to realize so called quantum simulators. Due to the controllability of atomic and optical systems, it could be useful to attack some unclear physics and to explore new phenomenon in quantum many-body systems [1–4]. In particular, over the past years the experimental progresses in controlling optical system [5–8] and in fabricating large-scale arrays of high-Q cavities [9, 10] make this potential application close to reality. However, the quantum optical system is typically driven by an external field and coupled to the environment [11, 12], which bring the system out of equilibrium and profoundly affected the dynamics of interest [13, 14]. New important questions thus arise and need to be clarified, such as whether the link between the initial ideas of cavity arrays as quantum simulators and the realistically experimental conditions is hold, and how dissipation and decoherence would behave in the system.

In this paper, we propose answers to these questions by investigating the superfluid(SF)-Mott insulator phase transition in the array of dissipative cavities. Provided the external time dependence is much slower than the internal frequencies of system, we show that there are still two fundamentally different quantum states, i.e. photons localized in each cavity(Mott-like) and delocalized cross the lattices(SF-like). However, a localized tendency does exist for both of these two states. Specifically, the ratio of the the intercavity coupling and on-site interaction strength for the transition from Mott-like region to SF-like region is modified by a time dependent increment. Moreover, the superfluidity of a initial SF state will break owing to the decay of the off-diagonal long-range order.

Consider a system consisted of atoms and cavities coupled weakly to a bosonic environment at zero temperature. As the dimension of individual cavities is generally much smaller than their spacing, we assume the photons emitted from each cavity are uncorrelated. The total Hamiltonian therefore reads

$$H = H_s + H_{bath} + H_{coup}. \tag{1}$$

where H_s is the Hamiltonian for the system, $H_{bath} = \sum_j \sum_{\alpha,k} \omega_{k_\alpha} r^\dagger_{j,k_\alpha} r_{j,k_\alpha}$ the Hamiltonian for environment, and $H_{coup} = \sum_j \sum_{\alpha,k} (\eta^*_{k_\alpha} r^\dagger_{j,k_\alpha} \alpha_j + h.c.)$ the coupled term. $\alpha = a, c$ labels the operators and physical quantities associated with atoms and cavities, respectively. ω_{k_α} denotes the frequency of environmental modes, $r^\dagger_{j,k_\alpha}$ and $r_{j,k_\alpha}$the creation and annihilation operators of quanta in the k_αth model at site j, and η_{k_α} the coupling strength. Here we set $\hbar = 1$.

The system we modelled, as depicted in Fig. 1, is a two-dimensional array of resonant optical cavities, each embedded with a two-level (artificial)atom coupled strongly to the cavity field. The possible realizations such as photonic bandgap cavities and superconducting stripline resonators et al. [4]. With ω_a and ω_c be-

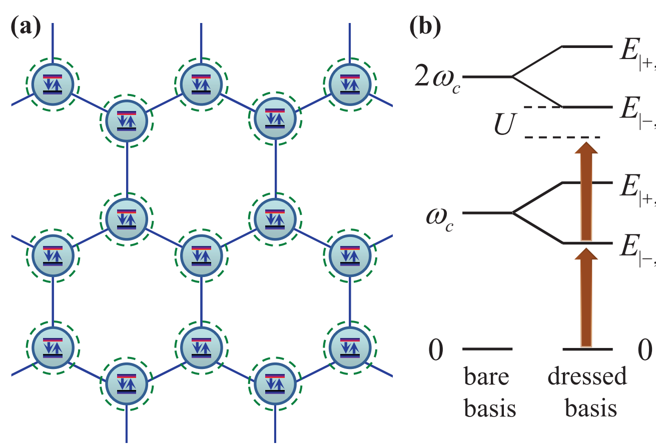

FIG. 1: A type of possible topologies for two-dimensional cavity arrays. (a) Individual cavities are coupled resonantly to each other due to the overlap of the evanescent fields. Each cavity contains a two-level system coupled strongly to the cavity field and immerses in a bosonic bath(marked by the dash line). (b) Energy eigenvalues of individual cavity-atom system on each site. $\omega_c = \omega_a$ is assumed for simplify. The anharmonicity of the Jaynes-Cummings energy levels can effectively provide an on-site repulsion U to block the absorption for the next photon.

ing the frequency of atom transition and cavity mode respectively, in the rotating wave approximation(RWA), such individual atom-cavity system on site j is well described by the Jaynes-Cummings Hamiltonian, $H_j^{JC} = \omega_a a_j^\dagger a_j + \omega_c c_j^\dagger c_j + \beta(a_j^\dagger c_j + h.c.)$. Here $a_j^\dagger$ and $a_j(c_j^\dagger,\, c_j)$ are atomic(photonic) rasing and lowering operators, respectively, β the coupled strength. In the grand canonical ensemble, H_s is therefore given by combing H_j^{JC} with photonic hopping term and chemical potential term,

$$H_s = \sum_j H_j^{JC} - \sum_{\langle j,j'\rangle} \kappa_{jj'} c_j^\dagger c_{j'} - \sum_j \mu n_j. \tag{2}$$

$\kappa_{jj'}$ is the photonic hopping rate between cavities. Since the evanescent coupling between cavities decreases with the distance exponentially, we restrict the summation $\sum_{\langle j,j'\rangle}$ running over the nearest-neighbors. $n_j = a_j^\dagger a_j + c_j^\dagger c_j$ is the total number of atomic and photonic excitations on site j. μ is the chemical potential, where the assumption $\mu = \mu_j$ for all sites has been made.

Due to the strongly coupling, as shown in Fig. 1(b), the resonant frequencies of individual atom-cavity system are split into

$$E_{|\pm,n\rangle} = n\omega_c \pm \sqrt{n\beta^2 + \frac{\Delta^2}{4}} - \frac{\Delta}{2}, \tag{3}$$

where $|\pm, n\rangle$ labels the positive(negative) branch of dressed states, $\Delta = \omega_c - \omega_a$ is the detuning. The anharmonicity of the Jaynes-Cummings energy levels can effectively provide a on-site repulsion. For instance, the resonant excitation by a photon with frequency $E_{|\pm,1\rangle}$ will prevent the absorption of a second photon at $E_{|\pm,1\rangle}$, which is the striking effect known as photon blockade [8]. It is therefore feasible to realize a quantum simulator in terms of the system described by Eq. (2). This so called Jaynes-Cummings-Hubbard(JCH) mode is recently suggested by Greentree et al. [2].

However, the situation changes dramatically once taking the coordinates of environment into consideration, as described by Hamiltonian(1). A non-equilibrium dynamics for open quantum many-body system do arise, which is a formidable task to solve. Here we propose a treatment to eliminate those external degrees of freedom. To approach this, we rewrite Hamiltonian(1) as

$$H = H_{local} - \sum_{\langle j,j'\rangle} \kappa_{jj'} c_j^\dagger c_{j'} - \sum_j \mu n_j, \tag{4}$$

where $H_{local} = \sum_j H_j^{JC} + H_{bath} + H_{coup}$.

First considering the case that the jth cavity contained a initial photon interacts with a bath, the dynamics is governed by

$$H_j = \omega_c c_j^\dagger c_j + \sum_k \omega_{k_c} r_{j,k_c}^\dagger r_{j,k_c} + \sum_k (\eta_{k_c}^* r_{j,k_c}^\dagger c_j + h.c.). \tag{5}$$

We denote its eigenvalue as ω and expand the eigenvector $|\phi_j\rangle$ as $|\phi_j\rangle = e_c c_j^\dagger|\emptyset\rangle + \sum_k e_k r_{k_c}^\dagger|\emptyset\rangle$. e_c and e_k are the probability amplitudes for the excitation occupied by cavity field and environment, respectively. $|\emptyset\rangle$ denotes the vacuum state. Deducing the equations of these two amplitudes, one can express e_k in terms of e_c and, under the Born-Markov approximation, integrate out the degrees of freedom of environment, and obtain

$$(\omega_c + \delta\omega_c - i\gamma_c)e_c = \omega e_c. \tag{6}$$

$\delta\omega_c$ is known as an analog to the Lamb shift in atomic physics and significantly small when the coupling to environment is weak. γ_c is the decay rate and indicates a finite lifetime of cavity mode [15].

This motivates us to introduce a quasi-boson described by C_j with a complex eigenfrequency $\Omega_c = \omega_c - i\gamma_c$, where $\delta\omega_c$ has been absorbed into ω_c, to effectively describe the open system described by Hamiltonian (5) in terms of

$$H_j^{eff}|\phi_j\rangle = \omega_j^{eff}|\phi_j\rangle, \tag{7}$$

with the effective Hamiltonian $H_j^{eff} = \Omega_c C_j^\dagger C_j$ and now $|\phi_j\rangle = e_c C_j^\dagger|\emptyset\rangle$. Because of loss, the system would be nonconservative and corresponding operators would be non-Hermitian. The communication relation of C_j reads $[C_j, C_{j'}^\dagger] = (1 + i\frac{\gamma_c}{\omega_c})\delta_{jj'}$. Recognizing $\frac{\gamma_c}{\omega_c}$ is in order of $\frac{1}{Q}$, with Q being the quality factor of individual cavity. The bosonic communication relation is therefore approximately satisfied for the high-Q cavity, which is meeting in most experiments about cavity quantum electrodynamics(QED).

The complex eigenfrequency underlines the facts that, on one hand, dissipation is the inherent property for realistic cavity. When a photon with certain frequency has been injected into a dissipative cavity, the composite system of cavity-filed plus environment cannot be characterized merely by the frequency of the injected photon, however, we must take the impacts of environment into account. On the other hand, in general we are not concerned the time evolution of bath. In this way, the array of dissipative cavities can be regarded as a configuration consisted of quasi-bosons. Quite similar operations can be performed on atom to introduce another kind of quasi-boson described by A_j with the frequency $\Omega_a = \omega_a - i\gamma_a$, where γ_a is the atomic decay rate.

We can therefore rephrase Hamiltonian (1) with the renormalized terms,

$$H = \sum_j H_j^{eff} - \sum_{<j,j'>} \kappa_{jj'} C_j^\dagger C_{j'} - \sum_j \mu n_j, \tag{8}$$

with now $H_j^{eff} = \Omega_a A_j^\dagger A_j + \Omega_c C_j^\dagger C_j + \beta(A_j^\dagger C_j + h.c.)$ and $n_j = A_j^\dagger A_j + C_j^\dagger C_j$. One key feature of Hamiltonian (8) is now the losses describe by leaky rates γ_a and γ_c but not by field oscillations. Without having to mention the external degrees of freedom, this effective treatment would be of great conceptual and, moreover, computational advantage rather than the general treatment as Hamiltonian (1). A more microcosmic consideration points out

that, in cavity QED region, since the atom is dressed by cavity field, the atom and field act as a whole subject to a total decay rate Γ [16]. In particular, $\Gamma = n(\gamma_a + \gamma_c)$ for $\Delta = 0$.

To gain insight over the role of dissipation in the SF-Mott phase transition, we use a mean field approximation which could give reliable results if the system is at least two-dimensional [17]. We introduce a superfluid parameter, $\psi = Re\langle B_j\rangle = Re\langle B_j^\dagger\rangle$. In the present case, the expected value of $B_j(B_j^\dagger)$ is in general complex with the formation $B_j = \psi - i\psi_\gamma(B_j^\dagger = \psi + i\psi_\gamma)$. ψ_γ is a solvable small quantity as a function of γ_a and γ_c, and vanishes in the limit of no loss. Using the decoupling approximation, $B_j^\dagger B_{j'} = \langle B_j^\dagger\rangle B_{j'} + \langle B_{j'}\rangle B_j^\dagger - \langle B_j^\dagger\rangle\langle B_j^\dagger\rangle$, the resulting mean-field Hamiltonian can be written as a sum over single sites,

$$H^{MF} = \sum_j \{H_j^{eff} - z\kappa\psi(B_j^\dagger + B_j) + z\kappa|\psi|^2 - \mu n_j + O(\psi_\lambda^2)\}, \tag{9}$$

where we have set the intercavity hopping rate $\kappa_{jj'} = \kappa$ for all nearest-neighbors with z labelling the number. Corresponding to the order parameter in related closed system, whether ψ vanishes or has a finite value can be used to identify the Mott-like and the SF-like regions.

ψ can be examined analytically in terms of second-order perturbation theory, with respect to the dressed basis. For energetically favorable we assume each site is prepared in the negative branch of dressed state. But because the dressed basis is defined on $n \geq 1$, a ground state $|0\rangle$ with the energy $E_{|0\rangle} = 0$ need to be supplemented,

$$\psi = e^{-\Gamma t}\sqrt{-\frac{\chi}{z\kappa\Theta}}. \tag{10}$$

χ and Θ are functions of all the parameters of the whole system. Since the evanescent parameter κ is a typical small quantity in systems of coupled cavities, the perturbation theory gives good qualitative and quantitative descriptions comparing to the numerically results given by explicitly diagonalizing [2, 18].

Arguably the most interesting situation is the photon-photon interactions are maximized, namely, cavities on resonant with atoms and with one initial excitations per site [19]. In addition, $\Gamma = \gamma_a + \gamma_c = \gamma$. With $F_1 = \omega_c - \beta - \mu$ and $F_2 = -\omega_c + (\sqrt{2} - 1)\beta + \mu$, in eq. (10), $\Theta = \frac{1}{2F_1^2+2\gamma^2} + \frac{3+2\sqrt{2}}{4F_2^2+4\gamma^2} > 0$, and

$$\chi = \frac{F_1}{2F_1^2 + 2\gamma^2} + \frac{(3+2\sqrt{2})F_2}{4F_2^2 + 4\gamma^2} + \frac{1}{z\kappa e^{-2\gamma t}}. \tag{11}$$

In the absence of loss, one can recognize $\chi = 0$ is the well known self-consistent equation and therefore distinguish the SF phase and Mott phase. Nevertheless, the coupling to environment inducing a non-equilibrium dynamics, and there is thus no strict phase exist. However, there remains a time-dependent boundary given by ψ which can distinguish two slowly decayed and qualitatively different regions, i.e. the SF-like and Mott-like regions.

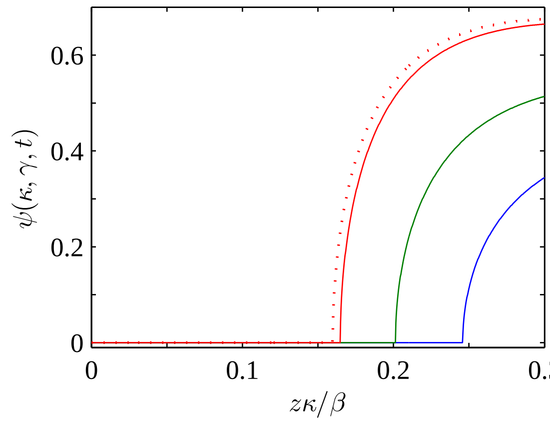

FIG. 2: The transition from Mott-like to SF-like region. Influences of dissipation on the critical ratio depend on the leaky rate γ (the dot and solid line for $\frac{\gamma}{\beta} = 0$, 0.05, respectively) and the time one reaches the transition (the red, green, and blue lines for $t = 0$, $0.1\gamma^{-1}$, $0.2\gamma^{-1}$, respectively).

In what follows, we discuss the physics of the transition between these regions from two aspects. First, we start in the Mott phase and discuss the impacts of dissipation on transition ratio $(\frac{z\kappa}{\beta})_c$. Consider the initial state is deep in the Mott phase, $\frac{z\kappa}{\beta} = 0$, and we continually increase the intercavity coupled rate. For the related ideal case, we will reach the SF phase at $\frac{z\kappa}{\beta} = (\frac{z\kappa}{\beta})'_c \simeq 0.16$. However, in the presence of dissipation, owing to the continuous leakage of coherence the effective tunnelling energy will be lower than what we set. Stated in another way, the photon hopping rate not only dependent on the coupling parameter between cavities, but also on the coherence inside cavities. As shown in Fig. 2, one cannot expect the appearance of photon hopping at $\frac{z\kappa}{\beta} \simeq 0.16$. We must continuously increase κ and the rate of increase must faster than the decay of coherence. Only in such way can the transition occur before the system reach the steady state. However, even we start exactly at $\frac{z\kappa}{\beta} = (\frac{z\kappa}{\beta})'_c$, we also cannot expect we are in the SF state. As the dissipation is the inherent nature of open quantum system, the critical ration $(\frac{z\kappa}{\beta})_c$, even at $t = 0$, is modified by an increment $\simeq \frac{2\gamma^2}{\beta^2}$. This can be understood in a simplified picture. If there was no excitation in system, i.e. we are in $|\emptyset\rangle$, the decay would not happen. However, the open quantum system is inherently different from the correspondingly perfected system.

In contrast, we start with the SF phase in the vicinity of transition ratio and track the time evolution of system. The exponential term in Eq.(10), $e^{-\Gamma t}$, indicates the expected decay of ψ and related physics quantities. However, the more important impacts of the presence of external environment are revealed by χ. As illustrated in Fig. 3, for $t \ll \beta^{-1}$, ψ has a slightly decrease scaled

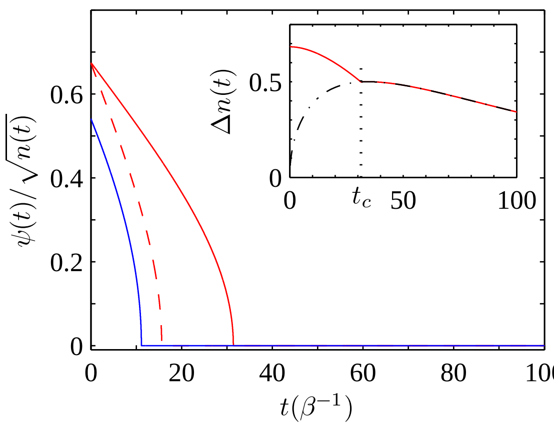

FIG. 3: The temporal decrease of the superfluidity and the photon number fluctuation on each site for a certain initial state(inset). For a initial SF state (the red line for $\frac{z\kappa}{\beta}=0.3$ and blue line for $\frac{z\kappa}{\beta}=0.2$), before t_c the long-range order decays continuously and the fluctuation on each site is a total effect of photon hopping and photon leakage (the solid line for $\frac{\gamma}{\beta}=0.01$ and dash line for $\frac{\gamma}{\beta}=0.02$). Beyond t_c, the superfluidity breaks down and the related photon number fluctuation behaves as the fluctuation of a Mott-like state (dot-dash line).

by $\frac{\gamma^2}{\beta^2}$. For $t>\beta^{-1}$, the leader term is $z\kappa e^{-2\gamma t}$. It is this term pronounces the decrease of effective tunnelling energy which, mathematically, originates from the non-diagonal element in Hamiltonian (9) respecting to the occupation number basis. Consequently, a photon hopping rate $\frac{z\kappa}{\beta}$ given initially in SF phase will cross the critical point at a time $t_c \simeq -\frac{1}{2\gamma}\ln\frac{\kappa}{\kappa_c'}$. Before t_c, a non-local region is still recognized as non-local. The dissipation does not change the fundamental nature of the system, albeit with the decrease of long-range coherence. Nevertheless, beyond t_c, the superfluidity breaks down and gives rise to a localized wavefunction, i.e. a transition to Mott-like region do occur. An analogous effect is recently described in optical lattice system, where a time-dependent tunneling rate controlled by external filed leads to a sweep from the SF to the Mott phase [20]. Differently, in our case the decay of long-range order is resulted from the leakage of coherence, and the total number of excitations is not conservative. As a result, there remains a statistical fluctuation acted on each site.

In conclusion, we have shown analytically the features of SF-Mott phase transition in the coupled cavity arrays in the presence of dissipation. Our analysis fully takes into account the intrinsically dissipative nature of open quantum many-body system, and identifies how dissipation and decoherence would come into play. For further experiment realization, we predict there is a localized tendency characterized by the suppression to the superfluidity.

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